

THE END RESULTS OF GENERAL THEORY OF RELATIVITY, VIA JUST ENERGY CONSERVATION AND QUANTUM MECHANICS

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Herein we present a whole new approach which leads to the end results of the General Theory of Relativity, via just the law of conservation of energy (broadened to embody the mass & energy equivalence of the special theory of relativity), and quantum mechanics. Thus, we start with the following postulate.

Postulate: The rest mass of an object bound to a celestial body amounts less than its rest mass measured in empty space, and this, as much as its binding energy vis-à-vis the gravitational field of concern.

The decreased rest mass, is further dilated by the Lorentz factor, if the object in hand, is in motion in the gravitational field of concern. The overall relativistic energy must be constant on a stationary trajectory. This yields the equation of motion driven by the celestial body of concern, via the relationship $e^{-\alpha} / \sqrt{1 - v_0^2/c_0^2} = \text{Constant}$, along with the definition $\alpha = GM/(rc_0^2)$; here \mathcal{M} is the mass of the celestial body creating the gravitational field of concern; G is the universal gravitational constant, measured in empty space; it comes into play in Newton's law of gravitation, which is assumed to be valid for static masses only; r points to the location picked on the trajectory of the motion, the center of \mathcal{M} being the origin of coordinates, as assessed by the distant observer; v_0 is the tangential velocity of the object at r ; c_0 is the ceiling of the speed of light in empty space; v_0 and c_0 remain the same for both the local observer and the distant observer, just the same way as that framed by the special theory of relativity.

The differentiation of the above relationship leads to

$$-\frac{GM}{r^2} \left(1 - \frac{v_0^2}{c_0^2}\right) = v_0 \frac{dv_0}{dr}, \text{ or via } v_0 = \frac{dr}{dt}, \quad -\frac{GM}{r^2} \left(1 - \frac{v_0^2}{c_0^2}\right) \frac{r}{r} = \frac{dv_0}{dt};$$

\underline{r} is the outward looking unit vector along r ; the latter differential equation is the classical Newton's Equation of Motion, were v_0 , negligible as compared to c_0 ; this equation is valid for any object, including a light photon.

Taking into account the quantum mechanical stretching of lengths due to the rest mass decrease in the gravitational field, the above equation can be transformed into an equation written in terms of the proper lengths, yielding well the end results of the General Theory of Relativity, though through a completely different set up.

Key words: Gravitation, Special Theory of Relativity, Mass Deficiency, Quantum Mechanics, Metric Change, General Equation of Motion, General Theory of Relativity

1. INTRODUCTION

This article, is essentially an extension of the controversial article which appeared in *Annales de La Fondation Louis de Broglie*, owing to a great referee's report [1]. Thus, we have achieved a whole new approach to the derivation of Newton's Equation of Motion, as well as the findings brought up within the frame of the general theory of relativity (GTR), such as the "precession of the perihelion of the planets", and the "deflection of light nearby a star".

To the contrary of what had been generally achieved so far, the basis adopted herein, merely consists in supposing that the gravitational field, via the binding process, alters the *rest mass* of an object conveyed in it. Indeed, the mass & energy equivalence drawn by the special theory of relativity (STR), astonishingly, far and wide overlooked, imposes such a change. Next to this fundamental theory, we use the classical Newton's gravitational attraction force reigning between two static masses. We have however, previously shown that, the $1/r^2$ dependency of the gravitational force is also imposed by the STR [2].

Furthermore, the metric coming into play in this work is (just like the one used by the GTR), altered by the gravitational field (in fact, by any field the “measurement unit” in hand interacts with); yet in the present approach, this occurs via quantum mechanics. In effect, the solution of even a non-relativistic quantum mechanical description, given that “*potential energies existing in nature*” are considered, leads to solutions, in perfect harmony with the STR. This is to say that, regarding the “*internal dynamics*” of a quantum mechanical object, “mass” (the mass, in fact, generally a compound mass, carrying this internal dynamics), “space” (i.e. the size of the object), and “time” (period of time displayed by the internal dynamics of concern), or “energy” (total energy marked out by this internal dynamics), are structured in such a way that, the quantum mechanical products, say [mass x size], and [size x period of time], are nailed to constants, made of the Planck Constant and electric charges, for atomic and molecular objects, for instance, which also remain Lorentz invariant (i.e. invariable, were the object brought into a uniform translational motion).

Thus, as we shall see, based on the STR, the rest mass of an object when embedded in a gravitational field should decrease as much as its binding energy in the field [3]; a mass *deficiency conversely*, via *quantum mechanics*, yields a stretching of its size, as well as the weakening of its internal energy (this is how the metric coming into play is altered by the field).

Einstein treats the effect of gravitation, via the assertion of the principle of equivalence [4]. Remarkably enough, we do not need this assertion. Thus, we come to show that, what the principle of equivalence does, can somewhat be accounted by the law of conservation of energy (broadened to embody the mass & energy equivalence of the STR). To show how this happens and demonstrate it, in a one to one correspondence, mathematically, constitutes the topic of a subsequent article.

Yilmaz, was one, who was disturbed by the manner the GTR, handled the rest masses, instead of envisaging the conservation of the total energy. In effect, once he derived the *exact solution* of the *accelerated elevator*, to his great surprise, he found out that Einstein’s fields equations were not satisfied. This was the beginning of his efforts towards a more consistent theory, though still based on the principle of equivalence [5,6,7]. Nonetheless, Yilmaz’s approach removed the singularities delineated by the GTR. Logunov, separately derived the relativistic theory of gravity, still removing, the singularities yielded by the GTR [8].

In any case, Einstein’s GTR leads to the fact that, his original relativistic “*mass & energy equivalence*”, obtained within the frame of the STR, i.e.

$$\begin{aligned} \Delta E \text{ (rest energy released, or piled up)} \\ = \Delta m \text{ (magnitude of the algebraic increase in the rest mass)} \times c_0^2 \end{aligned}$$

where c_0 is the ceiling of the speed, light in empty space can delineate, does not hold between values of energy and mass at different gravitational coordinates [9]. We do not have such an annoyance, since we derived our results essentially based on the above relationship.

It becomes amazing that Einstein’s relativistic “*mass & energy equivalence*” with regards to a rest mass embedded in a gravitational field, does wholly play the role of his extra assertion, i.e. *the principle of equivalence*, with regards to the effect of gravitation on a *rest mass*. In fact the relativistic mass & energy equivalence does even more, since the way we make usage of it, it can be applied to any field, the object in hand can interact with.

Thence one can propose the following postulate, actually nothing else, but the energy conservation law, where though, now *energy* and *mass* are essentially no different from each other.

Postulate: The rest mass of an object bound to a celestial body, amounts less than its rest mass measured in empty space, and this as much as its binding energy vis-à-vis the gravitational field of concern.

It is important to note that, on the contrary to what, the principle of equivalence suggests, in conjunction with the mass dilation, an accelerated object is to exhibit, here it is question of a *decrease* of rest mass in a gravitational field, and this is, interestingly, just as much as the rest mass increase, predicted by the GTR. Thus based on our approach, the classical red shift due to gravitation is nothing else, but a *rest mass decrease*, thereby a corresponding overall rest relativistic energy decrease of the emitter embedded in the gravitational field, whereas this is due to the clock retardation, within the frame of the GTR. (In our approach however, as we will soon see, the “*clock retardation*” is a phenomenon resulting from the “*rest mass decrease*”, caused by the gravitational binding, given rise to a quantum mechanical total energy decrease of the object in hand.)

Below, we first summarize our previous work [1] (Section 2). Next, we consider the *quantum mechanical stretching of units lengths*, due to the rest mass decrease, the object in hand displays in a gravitational field (Section 3). We thus land at a whole new general equation of motion yielding well the end results of the GTR (Sections 4 and 5). A conclusion follows (Section 6). It is worth to mention that the present work treats a light photon just like any other object.

2. PREVIOUS SET UP

The above postulate says that, the law of energy conservation must hold, through the gravitational binding process. It holds, in fact, through any binding process, driven by any possible force, the object in hand, may react to. Thus a particle at rest, when embedded in a given field (it interacts with), must *discharge* an amount of energy equal to the *binding energy* coming into play. Likewise, as the bound particle is carried out of the field in consideration, it will pile up, the amount of energy equal to its binding energy. Here, for simplicity, though without any loss of generality, we assumed that the bound particle is insignificant as compared to the host object binding it. We will handle this point below.

A mass deficiency conversely, via quantum mechanics (whose basis, i.e. the wave equation, together with, substantially, the de Broglie relationship, is already fully consistent with the STR), yields the *uniform stretching of the size* of the object at hand (in contrast with the contraction of the length of it coinciding with the direction of the gravitational attraction), as well as the *weakening* of its internal energy, via the following *quantum mechanical theorem* proven elsewhere [10,11,12,13].

Theorem 1: Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description points to an *internal dynamics* which consists in a “*clock motion*”, achieved in a “*clock space*”, along with a “*unit period of time*”. The description excludes “*synthetic potential energies*” (which may otherwise lead to incompatibilities with the STR). It is supposed to be based on \mathcal{K} particles, altogether. If then different masses m_{k0} , $k = 1, \dots, \mathcal{K}$, involved by this description of the object at rest, are all multiplied by the arbitrary number γ , the following two general results are conjointly obtained:

- a) The total energy E_0 associated with the given clock’s motion of the object is increased as much, or the same, the unit period of time T_0 , of the motion associated with this energy, is decreased as much.
- b) The *characteristic length*, or the *size* \mathcal{R}_0 to be associated with the given clock’s motion of concern, contracts as much.

In mathematical words this is:

$$[(m_{k0}, k = 1, \dots, \mathcal{K}) \rightarrow (\gamma m_{k0}, k = 1, \dots, \mathcal{K})] \Rightarrow [(E_0 \rightarrow \gamma E_0) \text{ or } (T_0 \rightarrow \frac{T_0}{\gamma}), \text{ and } (\mathcal{R}_0 \rightarrow \frac{\mathcal{R}_0}{\gamma})].$$

This, together with the above postulate, yields at once the next theorem.

Theorem 2: A clock interacting with any field, electric, nuclear, gravitational, or else (without loosing its “identity”), retards via quantum mechanics, due to the mass deficiency it develops in there, and this, as much as the binding energy it displays in the field in consideration; at the same time and for the same reason, the space size in which it is installed, stretches as much.

This can further be grasped rather easily, as follows. The mass deficiency, the quantum mechanical object displays in the gravitational field (or in fact, any field with which it interacts), weakens its internal dynamics as much, which makes it slow down. Thence, one arrives at the principal results, stated above.

In order to calculate the binding energy of concern, we make use of the classical Newton’s gravitational attraction force, yet with the restriction that, it can only be considered for *static masses*. Luckily we are able to derive the $1/r^2$ dependency of the “*Newton’s gravitational attraction force*” reigning between “*two static masses*”, here again, based on just the STR [2]. This can be achieved easily by noting that the quantity [force] x [mass] x [distance]³ is Lorentz invariant. On the other hand, it is empirically known that the electric charges are Lorentz invariant. Now suppose we have a “dipole” of a given mass at rest, bearing a given length r at rest. Coulomb Force reigns between the electric charges. Suppose we assume that Coulomb Force is, as usual, expressed as proportional to the electric charges coming into consideration, also to $1/r^n$, where though we do not know, a priori the exponent n . Suppose then we bring the dipole to a uniform translational motion, along the direction delineated by the line connecting the electric charges making it. Since for the case in consideration, [mass] x [length] remains invariant, it becomes evident that the Lorentz invariance of [force] x [mass] x [distance]³ shall hold, only if Coulomb Force, spatially behaves as [charge]² / r^n , but n being exclusively 2 (since charges are Lorentz invariant).

Note that the same holds, if charges in question, are “*gravitational charges*”; in this case however, the product of charges should be considered together with the universal gravitational constant. The foregoing reasoning, in this latter case, makes that the $1/r^2$ dependency of the Newton’s gravitational attraction force reigning, though between static masses only, is a requirement imposed by the STR.

Thus, in effect, our framework is fundamentally based on the STR.

The related metric (just like the one used by the GTR) is altered by the gravitational field (in fact, by any field the object in hand interacts with); though in the present approach, this occurs, as Theorem 1 underlines, via *quantum mechanics*. Henceforth, one does not require the *principle of equivalence* assumed by the General Theory of Relativity, as a precept, in order to predict the end results of this theory.

Let then $m_{0\infty}$ be the *rest mass* of the object in consideration, at infinity. When it is bound still at rest, to a celestial body of mass \mathcal{M} , assumed for simplicity infinitely large as compared to $m_{0\infty}$, this latter will be diminished as much as the binding energy coming into play, to become $m(r)$ (r being the distance of $m_{0\infty}$ to the center of \mathcal{M}), so that [1]

$$m(r) = m_{0\infty} e^{-\alpha(r)}, \quad (1)$$

where $\alpha(r)$ is

$$\alpha(r) = \frac{GM}{rc_0^2} ; \quad (2)$$

G is the universal gravitational constant; r is the distance of $m(r)$ to the center of \mathcal{M} , as assessed by the distant observer.

Note that $m(r)$ becomes the “*gravitational mass*”, if the object remains at rest. Otherwise, classically speaking, it is neither the “*gravitational mass*”, nor the “*inertial mass*”; it is the “*rest mass of the gravitationally bound object*”. This will be clarified at the level of Theorem 3, stated below. On the other hand, the reason for which, energy should be retrieved from the mass of the tiny bound object, and not from the infinitely more massive celestial body hosting it, is a requirement imposed by the law of conservation of linear momentum, which in return remains a result drawn by the law of conservation of energy and the Newton’s law of gravitational attraction force, we lie on. (Unfortunately, due to lack of space, we are to omit the mathematical details of the process in question.)

We would like to recall that, G is not Lorentz invariant, though classified as a universal constant. [One can immediately see this, as follows. Dimensionally speaking $GMm(r)$ is, in CGS unit system, equivalent to (electric charge)². But the electric charges are Lorentz invariant Thus, so must be the quantity $GMm(r)$. Mass is not a Lorentz invariant quantity. Hence neither G can be, though the product $GMm(r)$, is.] This shows that, G is not as universal as, one may think it is. Just like the mass, it is to depend on the gravitational field strength. This fact was coined in our previous article (Ref. 1), but left to be worked out.

Via differentiating Eq.(1), along with Eq.(2), it can be checked that we are indeed dealing with nothing else, but an *energy conservation equation*, i.e.

$$dm(r)c_0^2 = G \frac{m(r)\mathcal{M}}{r^2} dr . \quad (3)$$

In other words, the RHS of this equation is the *energy*, one would have to furnish to $m(r)$ at r , in order to carry it away from \mathcal{M} , as much as dr . And the LHS is the *energy equivalent of the mass increase* $dm(r)$, the mass $m(r)$ delineates throughout, as imposed by the STR. “*Energy conservation*” imposes that these two quantities are equal to each other (c.q.f.d.). We would like to recall that originally, we had arrived at Eq.(1), through a different set up than that underlined by Eq.(3).

Now suppose that the object of concern, say a planet, is in a given motion around the celestial body of mass \mathcal{M} , say the sun; the motion in question, can be conceived as made of two steps:

- i) Bring the object *quasistatically*, from infinity to a given location r , on its orbit, but keep it there *at rest*.
- ii) Deliver to the object at the given location, its motion on the given orbit.

The first step yields a decrease in the mass of $m_{0\infty}$ as delineated by Eq.(1). The second step yields the Lorentz dilation of the rest mass $m(r)$ at r , so that the overall mass $m_\gamma(r)$, or the same, the total relativistic energy of the object in orbit becomes

$$m_\gamma(r)c_0^2 = \frac{m(r)c_0^2}{\sqrt{1 - v_0^2/c_0^2}} = m_{0\infty}c_0^2 \frac{e^{-\alpha(r)}}{\sqrt{1 - v_0^2/c_0^2}} ; \quad (4)$$

v_0 is the “local tangential velocity” of the object at r .

The total energy of the object in orbit [i.e. $m_\gamma(r)c_0^2$] must remain constant [4, 14], so that for the motion of the object on a given orbit, one finally has *

$$m_\gamma(r)c_0^2 = m_{0\infty}c_0^2 \frac{e^{-\alpha}}{\sqrt{1-v_0^2/c_0^2}} = m_{0\infty}c_0^2 = \mathcal{D} = \text{Constant} \quad ; \quad (5)$$

(present basis of the equation of gravitational motion)

v_0 (the relative velocity of $m_{0\infty}$ in regards to \mathcal{M}), and c_0 , in our approach, if written in terms of lengths and periods of time picked up along the trajectories in consideration, remain the same for both the local observer and the distant observer (given that both, lengths and periods of time, are affected in the same respect), similarly to what is framed by the STR; \mathcal{D} is a constant to be determined.

3. WORKING OUT THE QUANTUM MECHANICAL STRETCHING OF LENGTHS

The differentiation of the above equation leads to

$$-\frac{GM}{r^2} \left(1 - \frac{v_0^2}{c_0^2} \right) dr = v_0 dv_0 \quad . \quad (6)$$

This relationship is interesting in many ways. First of all when v_0 (as compared to the velocity of light) is negligible, or similarly when α is small, it reduces right away to the classical Newton's equation of gravitational motion (which can be quickly checked for a free fall, for instance). This, interestingly means that we have come to modify the gravitational law of force. Eq.(6), as we will elaborate on below, can further account for the precession of the perihelion of the planets, as well as the deflection of light nearby a celestial body, were it processed with the content of Theorem 1 (though, it is obtained through a totally different perspective than that of Einstein). For the purpose in question, based on Eq.(1), we should make use of the following relationships, right away induced by Theorem 1, regarding how the proper period of time T_∞^\dagger and, the proper size \mathcal{R}_∞ , are *quantum mechanically* altered, in the gravitational field of concern, to become respectively $T(r)$ and $\mathcal{R}(r)$, as assessed by the distant observer:

$$T(r) = T_\infty e^\alpha \quad , \quad (7)$$

*(unit period of time at r, as assessed
by the distant observer)*

* Amazingly the GTR predicts (as furnished by Ref. 14)

$$m_\gamma(r)c_0^2 = m_{0\infty}c_0^2 \frac{\sqrt{1-2\alpha}}{\sqrt{1-v_0^2/c_0^2}} = \text{Constant} \quad (\text{relationship presented by Landau and Lifshitz}), \quad (i)$$

which coincides, up to the second order of the corresponding Taylor expansion, with Eq.(5); recall though, Eq.(5) (of the text), is fully consistent with what Yilmaz would have written, in the same way as that presented by Landau and Lifshitz, leading to Eq.(i), along with the correction (Refs. 5, and 6) he proposed to Einstein's metric, i.e.

$$m_\gamma(r)c_0^2 = m_{0\infty}c_0^2 \frac{e^{-\alpha}}{\sqrt{1-v_0^2/c_0^2}} = \text{Constant} \quad (\text{relationship written via Yilmaz' s approach}); \quad (ii)$$

† Note that the GTR predicts $T(r) = T_\infty / \sqrt{1-2\alpha}$. This outcome, to the first approximation, is the same as that of Eq. (7) of the text. Recall that because $T(r)$ is longer than T_∞ , the time registered by the distant observer in relation to the location r , runs slower than his own time, in empty space.

$$\mathcal{R}(r) = \mathcal{R}_\infty e^\alpha . \quad (8)$$

*(unit length at r as assessed
by the distant observer)*

Based on Eq.(8), it can be quickly shown that the perimeter P_0 of a circumference located at a distance r_0 of the local observer, from the center of the celestial body of mass \mathcal{M} , due to gravitation, stretches to become

$$P = P_0 e^\alpha \cong P_0 e^{\alpha_0} , \quad (9)$$

as measured by the distant observer. The measurement of r_0 , can be for instance achieved, via counting the local period of time, a light beam takes to make a round trip to \mathcal{M} .

Thus, r_0 shall be defined from

$$r = r_0 e^\alpha \cong r_0 e^{\alpha_0} ; \quad (10)$$

this makes that

$$dr \cong dr_0 e^{\alpha_0} + r_0 d\alpha_0 e^{\alpha_0} \quad (\text{for } \alpha_0 \ll 1) . \quad (11)$$

Recall that, r_0 is measured in terms of the local unit length [cf. Eq.(8)]. When Eqs. (10) and (11) are plugged into Eq.(6), one obtains

$$-\frac{G\mathcal{M}}{r_0^2} (1 - \alpha_0) e^{-\alpha_0} \left(1 - \frac{v_0^2}{c_0^2} \right) dr_0 = v_0 dv_0 . \quad (12)$$

*(approximate equation of motion written
by the author, in terms of the proper metric)*

The singularity for $\alpha_0 = 1$, arising in the LHS of Eq.(12), is due to the adoption of the approximation $r \cong r_0 e^{\alpha_0}$, instead of the rigorous relationship, $r = r_0 e^\alpha$ [cf. Eq.(10)].

The correct equation of motion must have been

$$-\frac{G\mathcal{M}}{r_0^2} \left(\frac{e^{-\alpha}}{1 + \alpha} \right) \left(1 - \frac{v_0^2}{c_0^2} \right) dr_0 = v_0 dv_0 , \quad (13)$$

(written rigorously by the author, in terms of the proper metric)

instead of Eq.(12).[‡]

[‡] The relationship $r = r_0 e^\alpha$ indeed yields

$$dr = \frac{dr_0 e^\alpha}{1 + \alpha} , \quad (i)$$

or

$$\frac{dr}{r^2} = \left(\frac{e^{-\alpha}}{1 + \alpha} \right) \frac{dr_0}{r_0^2} , \quad (ii)$$

to be plugged in the LHS of Eq.(6). On the other hand, α can be extracted from

$$\alpha = \alpha_0 e^{-\alpha} , \quad (iii)$$

or the same, from $\alpha e^\alpha = \alpha_0$, in terms of α_0 .

Note that Eq.(13), via Eq.(10), becomes[§]

$$\frac{\exp(-\alpha_0 e^{-\alpha_0})}{\sqrt{1-v_0^2/c_0^2}} \cong \mathcal{D}_0, \quad (14)$$

\mathcal{D}_0 still being a constant, but now as assessed by the local observer. Eq.(14), evidently, is the integral form of Eq.(13).

4. GENERAL EQUATION OF GRAVITATIONAL MOTION IN VECTOR FORM

In order to draw a one to one comparison between the classical Newtonian frame [15], and ours, we would like to rewrite Eq.(12), by differentiating Eq.(14), and reexamine it:

$$\frac{G\mathcal{M}e^{-\alpha_0}(1-\alpha_0)}{r_0^2} m_{0\infty} \exp(-\alpha_0 e^{-\alpha_0}) dr_0 = \frac{m_{0\infty} \exp(-\alpha_0 e^{-\alpha_0}) \frac{-v_0}{\sqrt{1-v_0^2/c_0^2}} dv_0}{\sqrt{1-v_0^2/c_0^2}}. \quad (15)$$

[Eq.(12), rewritten by differentiating Eq.(14)]

The LHS of this equation expresses the infinitesimal change in the static gravitational binding energy of the object, making that the overall mass to be accounted by the static gravitational force exerted on the object becomes $m_{\infty} \exp(-\alpha_0 e^{-\alpha_0}) / \sqrt{1-v_0^2/c_0^2}$. The RHS conversely expresses the infinitesimal change in the kinetic energy of this “overall mass”. Note that the change on the kinetic energy, is solely due to the change on the velocity. Recall further that the infinitesimal changes in question are in opposite directions.

Eq.(15) is an equality between energies, and not obviously between forces, though as we will elaborate below, it can well be transformed into an equality between forces.

[§] Newton wrote for the *static gravitational force* F , between \mathcal{M} and m ,

$$F = G \frac{\mathcal{M}m}{r^2}. \quad (i)$$

(*gravitational force as assessed by the distant observer*)

We claim that, this expression is only valid as assessed by the distant observer, whose metric being the *only metric* unaffected by the presence of \mathcal{M} . Locally, say as referred to m , the distance r_0 of m to \mathcal{M} , as assessed by an observer situated at the location of m , is different from r ; more specifically, it is shorter than r , as much as e^{α_0} [cf. Eq.(10)]. Thus F is transformed into F_0 , i.e.

$$F_0 = G \frac{\mathcal{M}m e^{-2\alpha_0}}{r_0^2} \quad (\text{for } \alpha_0 \ll 1). \quad (ii)$$

(*gravitational force as assessed by the local observer*)

Recall that $G\mathcal{M}m$ remains invariant, regardless the coordinate system, just like electric charges embarked in a translational motion, are. (*This assertion is in full conformity with the Galilean principle of relativity, which is in return the essential pillar of the STR.*) The outcome is that, it is as if G is decreased by $e^{-2\alpha_0}$, making that Fdr is decreased by $e^{-\alpha_0}(1-\alpha_0)$ [cf. Eqs.(11) and (12)]. In effect, the scheme we have just drawn, is not any different from that displayed by the change of the law of Coulomb Force, exerted by moving electric charges, upon each other. To be more specific, let γ be the usual Lorentz dilation factor. Then the force intensity as observed by the outside fixed observer, is γ^2 times smaller than that of the proper force, due to the length contraction (*the charges being untouched*). In a gravitational field, lengths *stretch*, instead. Thus in this latter case, we expect just the opposite to occur. This result fully supports the internal coherence of the present approach.

Thence by rereading Eq.(15), we can state that (see Ref.1)

$$(Classical\ Gravitational\ Force)e^{-\alpha_0}\sqrt{1-v_0^2/c_0^2} = (Overall\ Mass) \times (Acceleration). \quad (16)$$

If the force is assessed by the distant observer, on the other hand, Eq.(16) becomes

$$(Classical\ Gravitational\ Force)\sqrt{1-v_0^2/c_0^2} = (Overall\ Mass) \times (Acceleration). \quad (17)$$

Eq.(12), indeed, following the routine pattern (used to transform the Newton equation, connecting the change in the binding energy with the change in the kinetic energy), can be transformed into a time dependent equation of motion [1,16,17], to write

$$-\frac{GM}{r_0^2}(1-\alpha_0)e^{-\alpha_0}\left(1-\frac{v_0^2}{c_0^2}\right)\underline{r}_0 = \frac{d\underline{r}_0^2(t_0)}{dt_0^2}; \quad (18)$$

(the general equation of gravitational motion written
by the author, in the local frame of reference)

here \underline{r}_0 is the vector bearing the magnitude r_0 , and directed outward; recall that $d\underline{r}_0/dt_0$ is the velocity vector \underline{v}_0 at time t_0 , the local time.

Taking into account the fact that, the universal gravitational constant decreases as much as $e^{-2\alpha}$, in the proper frame of reference [cf. the footnote we framed right above Eq.(14)], to become G_0 , i.e.

$$G_0 = G \exp(-2\alpha), \quad (19)$$

for a not too strong field, Eq.(18) can be written as

$$-\frac{G_0 M}{r_0^2}\left(1-\frac{v_0^2}{c_0^2}\right)\underline{r}_0 = \frac{d\underline{r}_0^2(t)}{dt_0^2}. \quad (20)$$

Eq.(16), based on the analysis of Eq.(14), seems the natural way of presenting our result. But then, Newton's equation of gravitational motion, i.e. [Newton's Gravitational Attraction Force = Mass of the Planet x Acceleration] is broken, since an extra term, i.e. $(1-\alpha_0)e^{-\alpha_0}\sqrt{1-v_0^2/c_0^2}$ comes to multiply the Newton's gravitational attraction force, in its classical form. The classical cast, nonetheless can formally be saved, if instead, we choose to alter the "classical Newton's gravitational attraction force"; but then the *gravitational mass* and the *inertial mass*, as classically defined, shall not be same. Thus, we establish our next theorem.

Theorem 3: The gravitational mass m_{0G} , and the inertial mass m_{0I} , as classically defined, are not the same; the theory presented herein, to formally save Newton's equation of gravitational motion, in reference to the local observer, predicts

$$m_{0G} = m_\infty \exp(-\alpha_0 e^{-\alpha_0}) \sqrt{1-v_0^2/c_0^2},$$

given that

$$m_{0I} = \frac{m_\infty \exp(-\alpha_0 e^{-\alpha_0})}{\sqrt{1-v_0^2/c_0^2}}.$$

Though very little, generally, m_{0G} and m_{0I} differ.

The above theorem yields the interesting relationship along with the usual definition of the Lorentz dilation factor:

$$m_{0G} = m_{0I} / \gamma^2. \quad (21)$$

Eq.(18), on the other hand, seems to be remarkable for velocities not negligible as compared to the velocity of light, since it indicates that in such a case, the effective attraction turns to be much divergent than the classically estimated one.^{**} In other words, one can deposit a satellite on a given stationary orbit, with a velocity lower than that predicted by the classical Newtonian set up. Or the same, the usual Newton's gravitational attraction force induces a stationary motion, which for a given velocity, takes place much closer to the gravitation source (in comparison with what would have classically occurred according to the Newtonian description). This is in fact, according to our approach, how the perihelion of a planet, precesses.

By the same token, a given orbit velocity now corresponds to a much larger source mass, at a given distance from this. This may constitute a clue to the dark matter quest.^{††} At the far edge of the universe, where the escape velocity of the objects is very high, the restoring gravitational force exerted by the rest of the universe on these objects, turns out to be much weaker than the classically estimated one. In other terms, the higher the escape velocity, the greater appears the acceleration to be classically coupled with the same mass of the universe residing under; this is as if the universe is pulled toward outside. This may constitute a clue to the dark energy quest.

^{**} Consider a stationary circular orbit; then based on Eq.(18), one can, to the first approximation, write

$$\alpha \left(1 - \frac{v_0^2}{c_0^2} \right) = \frac{v_0^2}{c_0^2} ; \quad (i)$$

α , following Eq.(10) is defined as

$$\alpha = \alpha(r) = \frac{GM}{c_0^2 r} . \quad (ii)$$

Eq.(i) leads to

$$\frac{v_0^2}{c_0^2} = \frac{\alpha}{1 + \alpha} , \quad (iii)$$

(within the frame of the present approach)

whereas within the frame of the Newtonian approach one has

$$\frac{v_0^2}{c_0^2} = \alpha . \quad (iv)$$

(within the frame of the Newtonian approach)

Thus, for a given α , the ratio v_0^2/c_0^2 predicted by the present approach, is smaller than that predicted by the Newtonian approach. Or the same, along Eq.(i), a given velocity yields an α greater than predicted by the Newtonian approach; this means a smaller orbit radius, or equivalently, a more massive gravitation source. Hence the closer v_0 to the speed of light, with respect to the circular motion in consideration, the nearer is α to infinity (instead of unity, within the frame of the Newtonian approach.) Or the same, $\alpha=1$ leads to $v_0=c_0$, in Newtonian mechanics, whereas $\alpha=1$ yields $v_0=c_0/\sqrt{2}$, following our approach.

^{††} Attention should be paid to avoid the following possible confusion. The masses of bound celestial bodies, though very little, decrease, and what they mutually sense, comes from the effectively decreased masses in question. Thus, it may seem confusing to mention the concept of "dark matter" at this point, while one seeks more matter than "observed" and not matter lost due to binding. Nonetheless Eq.(18) ultimately tells us that, less mass than usual yields higher stationary orbit velocity. It is that the law of force has changed, and what is reigning, is not anymore the usual Newtonian law of force, though it is based on this latter.

5. PRINCIPAL END RESULTS OF THE GTR

The essence of this work was based on the *energy conservation law*, including the *equivalence of mass and energy*, as implied by the STR. This led to the general equation of gravitational motion, i.e. Eq.(18). This latter, becomes the Newton's equation of motion, only if v_0 is small as compared to the velocity of light, and the gravitational field is weak. This equation strikingly, can cover up the basic predictions envisaged by the GTR, though its set up is totally different than that of this theory. Note that a comparable equation was assumed based on a Weber's law of electric force [18,19], though brutally transposed to celestial mechanics [20,21]. Thus, instead of the coefficient $(1 - \alpha_0)e^{-\alpha_0}(1 - v_0^2/c_0^2)$, coming into play in the LHS of Eq.(18), briefly speaking, the coefficient $(1 - 3v_0^2/c_0^2)$ was visualized, where though, the multiplier 3, next to v_0^2/c_0^2 , appears to be deprived of any explanation. This latter approach anyway happens to take care of the precession of the perihelion of Mercury, with the inconvenience of not being able to cover the bending of light. It is that this approach and Eq.(18), become the same, were α_0 and v_0^2/c_0^2 are small and equal to each other, which is the case for a weak gravitational field, driving a nearly circular motion regarding the bound object.

Precession of the Perihelion of Planets

One can quickly notice that, Eq.(18) accounts satisfactorily for the precession of the perihelion of Mercury. In this aim, for a change, we can merely compare the relative increases in the velocities, throughout the journey of say, Mercury along its trajectory, predicted by Newtonian approach, and the present theory. Thus, from Eq.(12) we have

$$\left. \frac{dv_0}{v_0} \right|_{\text{Author}} = \frac{c_0^2}{v_0^2} (1 - \alpha_0) e^{-\alpha_0} \left(1 - \frac{v_0^2}{c_0^2} \right) d\alpha_0, \quad (22)$$

versus the corresponding classical Newton formulation

$$\left. \frac{dv_0}{v_0} \right|_{\text{Newton}} = \frac{c_0^2}{v_0^2} d\alpha_0, \quad (23)$$

devoid of the extra multipliers coming into play, through the present approach.

As seen, the relative increase in the velocity, drawn by our approach [Eq.(22)], is a bit smaller than that drawn by the Newtonian approach [Eq.(23)]. Thus, it is as if, the Newtonian planet's velocity in question, has slowed down. Accordingly, the difference of the right hand sides of Eqs. (22) and (23) will, to a first approximation, provide us with the answer of how much the classical orbit, tilts toward the sun, around the perihelion. Thence, the precession \mathfrak{P}_p of the perihelion of the planet, in question (cf. Figure 1), through each revolution can then be expressed as

$$\mathfrak{P}_p \cong \oint \cot \text{an} \theta \left. \frac{dv_0}{v_0} \right|_{\text{Author}} - \oint \cot \text{an} \theta \left. \frac{dv_0}{v_0} \right|_{\text{Newton}} \cong - \oint \frac{c_0^2}{v_0^2} \left(2\alpha_0 + \frac{v_0^2}{c_0^2} \right) \cot \text{an} \theta d\alpha_0, \quad (24)$$

where, we approximated the exponential term with its first order Taylor expansion.

One can further evaluate the term v_0^2/c_0^2 , via using our original set up, i.e. Eq.(5), together with the expression of the constant \mathcal{D} , given by (see Ref. 1),

$$\mathcal{D} \cong 1 - \frac{GM}{2c_0^2 a} = 1 - \frac{\alpha(a)}{2}, \quad (25)$$

where a is the semi grand axis, of the planet's orbit.

Thus, we finally land at

$$\mathfrak{S}_P \cong -\oint \left(1 + \frac{2\alpha_0}{2\alpha_0 - \alpha(a)} \right) \cot \theta d\alpha_0. \quad (26)$$

One can notice that α_0 is very close to $\alpha(a)$ (and equal to this latter quantity, for a circular orbit), which makes that the integrand taking place in the above relationship becomes $1+2/1=3$; one can sense that this will hold even firmer, in the integral sense framed by Eq.(26). Thence, we can write^{††}

$$\mathfrak{S}_P \cong -\oint_{\text{orbit}} 3 \cot \theta d\alpha = 6\pi \frac{GMa}{b^2 c_0^2}, \quad (27)$$

(the rotation angle of the major axis of Mercury's orbit,

through each revolution, derived by the author)

which happens to be exactly the same result as that furnished by the GTR (though the corresponding descriptions, and the resulting equations are entirely different from each other).

Bending of Light

Note that Eqs. (22) and (23) stay valid for the light bending too. For light, v_0^2/c_0^2 is very close to unity throughout (since, although light gains energy while falling into the gravitational field, at the speed it cruises, its velocity is barely affected). One can then (via opening the orbit sketched in Figure 1 into a straight line, along the drawn tangent representing the light's trajectory), immediately calculate difference $\mathfrak{S}_{\text{extra}}$, between the results furnished by our approach and that of Newton. Thus we have

$$\mathfrak{S}_{\text{extra}} \cong -\int_{-\infty}^{\infty} \frac{c_0^2}{v_0^2} \left(2\alpha_0 + \frac{v_0^2}{c_0^2} \right) \cot \theta d\alpha_0 \cong -\int_{-\infty}^{\infty} \cot \theta d\alpha_0 = 2 \frac{GM}{Rc_0^2}, \quad (28)$$

(extra bending caused by a weak gravitational field,

to be accounted for, next to the classical Newtonian bending)

i.e. just as much as the Newtonian deflection, assuming that light grazes the celestial body of radius R , which makes that, the overall light bending comes to be twice as that predicted by the Newtonian set up; this result, holds for a weak gravitational field. Note that R , in Figure 1, is the minimum distance of the center of attraction to the tangent, representing the light's original trajectory.

Recall that, although the light velocity v_0 is very close to c_0 , in our approach it is not equal to this limit. (Only a light photon hypothetically bearing an infinite amount of energy, can reach the speed c_0 .) This makes that the case of a strong field should be handled separately. Nonetheless, via the comparison of Eq.(12) and Eq.(13), we can quickly predict that a strong field (as insignificant as this may be), reduces the bending.

6. CONCLUSION

This work is based on the fundamental law of energy conservation considered along with the mass & energy equivalence drawn by the special theory of relativity (STR). Thus

^{††} Special thanks are due to *Professor Elman Hasanov*, from Isik University, who kindly achieved the integrals in question.

herein it is question of the “*conservation of the overall relativistic energy*” delineated by a closed system.

The second ingredient we used, is the *Newton’s gravitational attraction force*, reigning between *static masses* exclusively. Yet we have shown that the spatial dependency of this force is imposed by the STR. The third ingredient we used, is *quantum mechanics*. The nature of this ingredient is nothing else, but energy conservation, together with de Broglie relationship, which in turn is fully compatible with the STR. Such a setup, based on our approach, eventually tells us, how a given object should be structured, already at rest, in order to ensure the end results of the STR, as well as those of the GTR [10,11,12,13]. This is what, in effect, Theorem 1 tells us.

Thus, we have indeed operated based on just the law of energy conservation, though broadened to embody the mass & energy equivalence of the STR, and nothing else. This is deep in many ways. First of all we were successful to obtain all of the major end results of the general theory of relativity (GTR), i.e. red shift, precession of the perihelion of a planet, and light bending (up to second order of a corresponding Taylor expansion), through though, a *completely different setup* than that of the GTR. Note that we have reached, astonishingly, *no singularity*, and further have treated light just like an ordinary object. A photon falling in a gravitational field gains energy, though its velocity does not significantly increase. (It does increase anyway, and only a photon of infinite energy would bear the ceiling speed, we denote by c_0 .)


How is it that we obtained virtually the same results as those of the GTR? According to our approach, matter is *quantum mechanically* built to yield such results. More specifically, the rest *relativistic energy* of an object embedded in a gravitational field (or in fact any field the object may interact with), is decreased, owing to the *law of conservation of energy*, as much as the *binding energy* coming into play. If so, then owing to the *mass & energy equivalence* of the STR, its *rest mass* is decreased. Thereby (via solely the quantum mechanical description of the object in hand), the gravitational red shift occurs (Theorems 1 and 2). If the object is in motion in the field, its relativistic energy is further altered by the corresponding Lorentz factor, and one well gets to a general equation of motion, governing the destiny of the object, whether this is an *ordinary object* or a *photon* [Eqs. (5), (6), 12, and (13)].

This is fascinating, but puts the *principle of equivalence* (PE) of the GTR at stake as well. What shall we do with it? Sincerely this is not our problem. Yet, clearly we have arrived at different expressions for *gravitational mass* and *inertial mass* (cf. Theorem 3). Because of this, we happen to attract conservative reactions, and we have to say something about the PE. The reader may refer to the Ref.1, where we have scrutinized this problem. But the basic issue is that, very many authors misinterpret this principle. So: i) We severely have a definition problem. ii) What the PE envisages, is a *rest mass* sitting in a gravitational field, in comparison with the same mass sitting (*at rest*) on the floor of an accelerating elevator, but not really a *mass in motion* in a gravitational field.

Thus Newton’s, classical equation of motion is really *not the grounds* for comparison of the *gravitational mass* entering in the expression of *Newton’s gravitational attraction force* with the *inertial mass* entering in the expression of *Newton’s law of motion* (since the latter mass is evidently not *resting* in the gravitational field of concern, but it is in motion). Nonetheless, Newton was quite right to question whether his two masses are equal or not, and we have shown that they are not (cf. Theorem 3). Fortunately for the sake of the GTR, what we show by Theorem 3, is not really the breaking of the PE. But, we could arrive to the end result of the GTR concerning the effect of gravitation vis-à-vis an object at rest, via simply *energy conservation*. And this is of course tricky. In other words, when things are simplified,

what Einstein calculates via the PE, is the numerator of the RHS of Eq.(4), and we have practically the same result as his [cf. Eq.(4), and the footnote below Eq.(5)].

Thus amazingly, we come to show that, what the *principle of equivalence* does, can well be accounted by the *law of conservation of energy (broadened to embody the mass & energy equivalence of the STR)*, and *quantum mechanics (which is in full compatibility with the STR)*. How can this be? Well, *one answer* is what we have done through this paper. Otherwise, the answer is surely profound, and constitutes the topic of a subsequent article, where we propose to demonstrate mathematically how the PE and energy conservation together with the mass and energy equivalence of the STR, could lead to *alike results*, and this happens to be not complicated at all. It is that the PE consists in, unfortunately, a non-conform analogy" between the effect of acceleration and the effect of gravitation. The remedial of the non-conformity in question, astoundingly, leaves the PE unnecessary!

What we did herein, in any case, is the removal of the blockade between different packages designed to conceive different scales of nature. Now both atoms and celestial bodies can be modeled along with the same approach, and gravitation can be quantized [22]. Along this line, we have to note that our approach, well leads to de Broglie relationship, for both electrically and gravitationally interacting objects. 

Note on the other hand that, recently V. Andreev reported to the PIRT Conference, held in July 2005 in Moscow, that a pendant load irradiated at the General Physics Institute of the Russian Academy of Sciences, by high energy electrons, comes to weigh less than its untouched twin counterpart [23]. The author of this article, following Andreev's presentation, suggested that, the effect must be due to energizing the unpaired electrons of the atoms of the load in consideration (which happened to be duraluminium); these electrons, based on Theorem 3 and the succeeding Eq.(21), become practically weightless. A quick calculation indeed proves this point of view, which shall be elaborated in a subsequent article.

Our approach further, unlike the GTR, is not restricted to gravitation. Thus a bound muon's decay rate for instance, should retard, just like the hydrogen's light is red shifted in a gravitational field [24].

Nature being one, we should be able to conceive it, with just one model. We hope that, we were able to present herein a clue, toward such a goal.

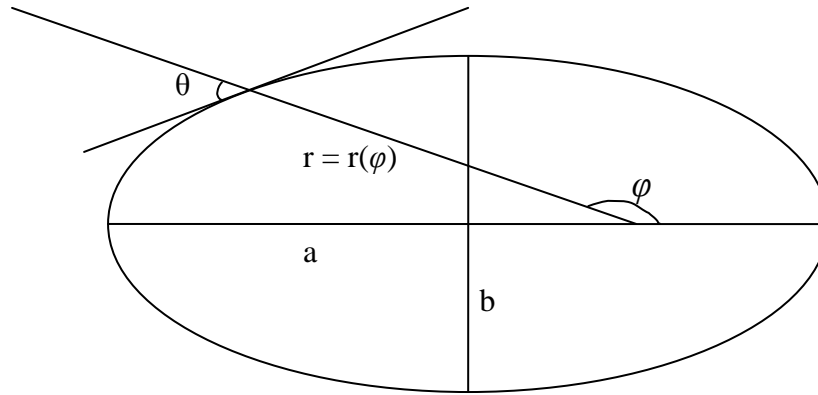


Figure 1 Mercury's orbit

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